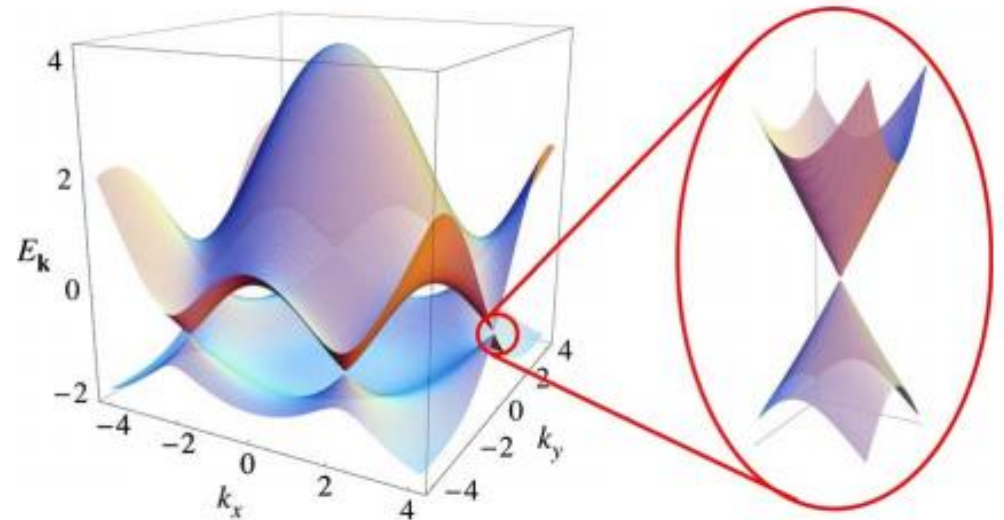


# Fantastic and Physically Appealing: the Electronic Structure and Properties of Graphene

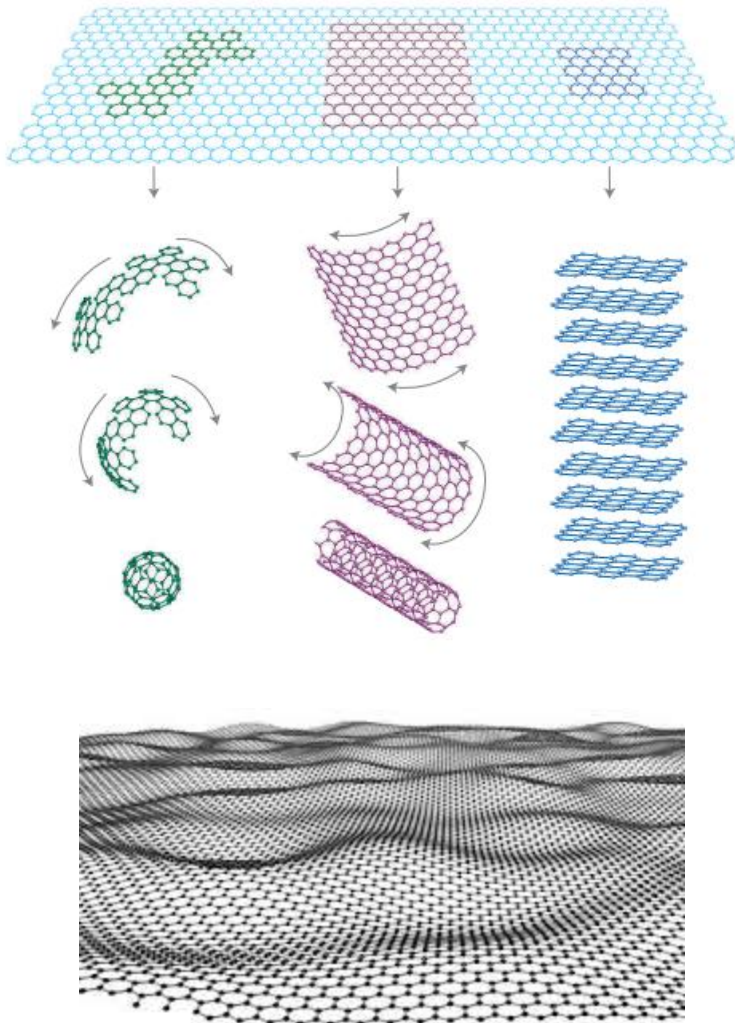
燕保明 (Speaker) 章亮、胡蕊、李彩珍、牛晶晶、周旭、柳晨、杨传剑、  
汤宏建、陈志超



# Outline

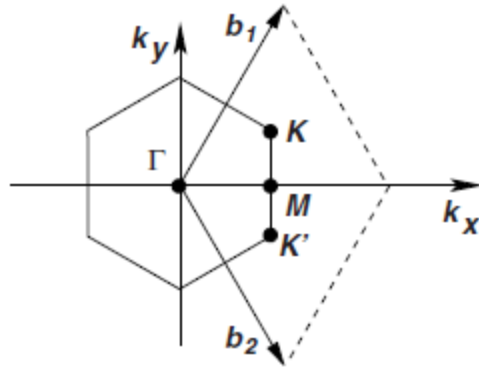
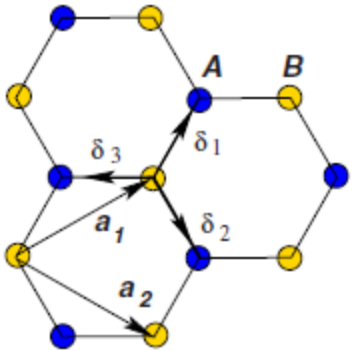
- **Elementary Introduction**
- **Electronic Structure**
- **Disorder of Graphene**
- **Transport (Near and far from Dirac Cone)**
- **Quantum Hall Effect**
- **Summary**

# Elementary Introduction



- **Strictly 2D crystals were thermodynamically unstable and could not exist.**(Landau& Peierls)
- **Theoretical study: Electronic structure**(Philip Wallace,1947) name ‘graphene’(S.Mouras,1987)
- **Experiment seeking:**  
**Extract by AFM (P.Kim Group, Columbia University)**  
**Epitaxy on SiC (Walt A. de Heer Group,Georgia Institute of Technology)**
- **Success:**  
**Scotch tape technique**(Andre Geim and Kostya Novoselov, Manchester University)  
**Nobel Prize 2010!**
- **Ripples made graphene possible**  
**(~1nm height ,~10nm in planar)**

# Electronic Structure



Above left: lattice structure of graphene  
 Above right: corresponding Brillouin zone.  
 Below: Band structure of graphene

Band (Tight-binding approach)

$$E_{\pm}(\mathbf{q}) \approx 3t' \pm v_F |\mathbf{q}| - \left( \frac{9t'a^2}{4} \pm \frac{3ta^2}{8} \sin(3\theta_{\mathbf{q}}) \right) |\mathbf{q}|^2,$$

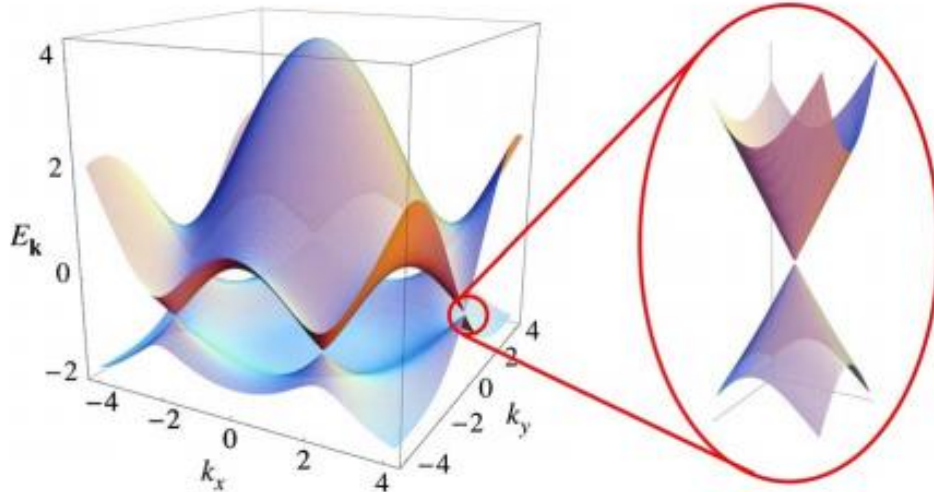
where

$$\theta_{\mathbf{q}} = \arctan\left(\frac{q_x}{q_y}\right)$$

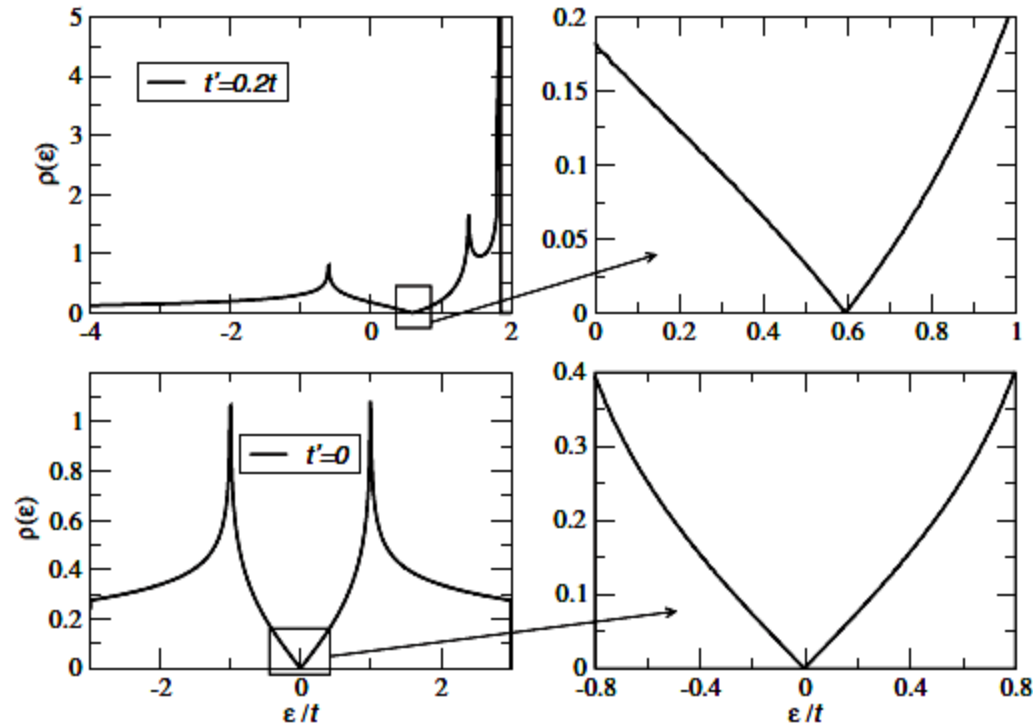
Considering  $t (\approx 2.5 \text{ eV}) \gg t' (\approx 0.1 \text{ eV})$

$$E_{\pm}(q) = \pm \hbar v_F q + O(q/k)^2$$

$\mathbf{q}$  is the momentum measured relatively to the Dirac points and  $v_F$  is the Fermi velocity, with a value  $v_F \approx 10^6 \text{ m/s}$  ( $c/300$ )



# Density of States

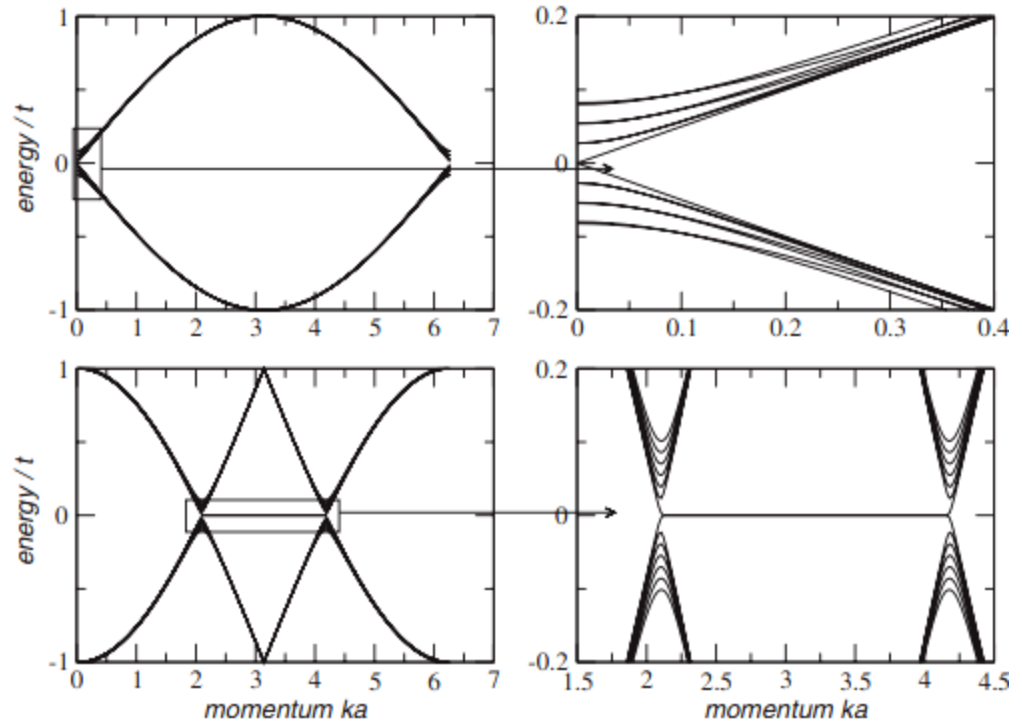
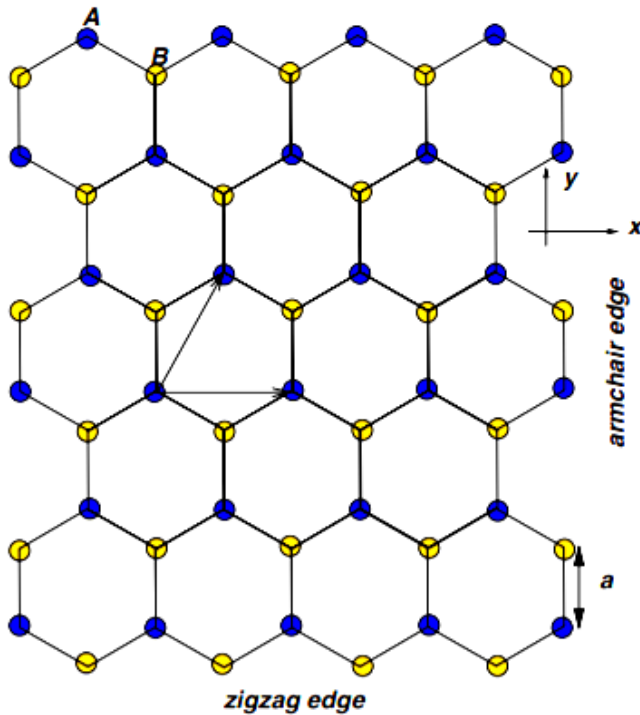


- Considering the degeneracy is four, (spin&valley),

$$\rho(E) = \frac{2A_c |E|}{\pi v_F^2},$$

where  $A_c$  is the unit cell area given by  $A_c = 3\sqrt{3}a^2/2$ .

# Surface(Edge) State & Nanoribbon

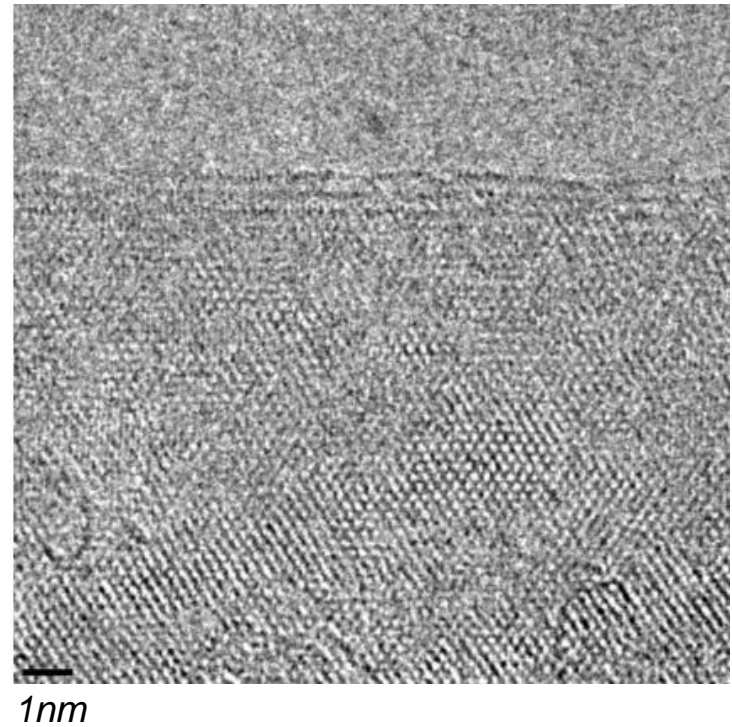
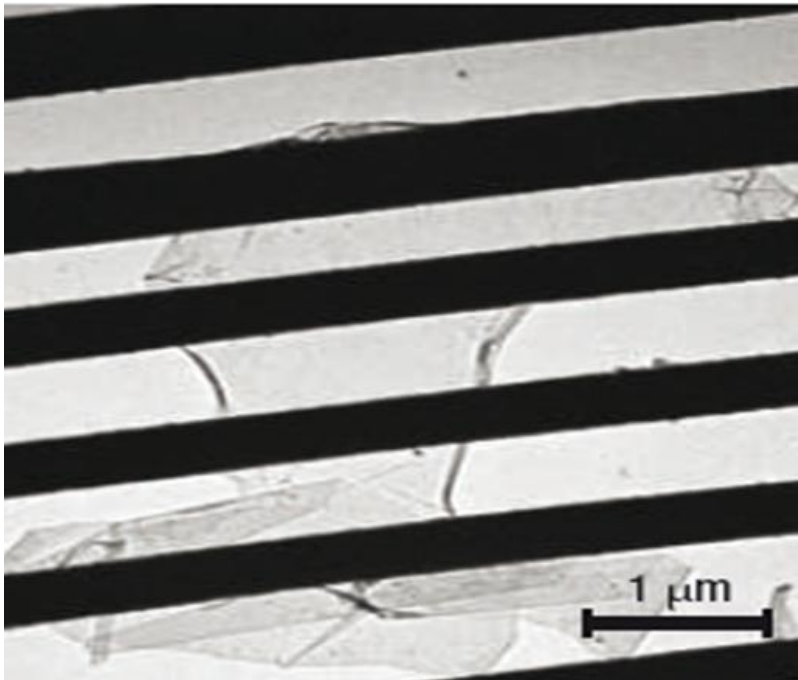


- **Armchair nanoribbons:**  
zero or no-zero gap , based on width( $N$ ), no edge state
- **Zigzag nanoribbons:**  
zero gap, exist localized edge state(near fermion energy)



# Disorder in Graphene

- Ripples
- Topological defects
- Impurities
- Ad-atoms
- Cracks
- Edges



# Why Ripples

- **Number of flexural modes per unit of area at a certain temperature T**

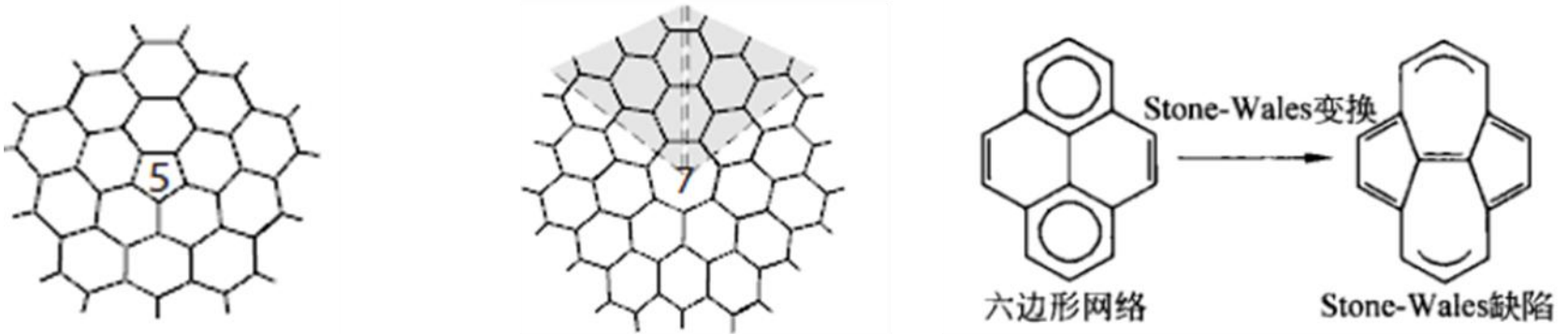
$$N_{\text{ph}} \approx \frac{2\pi}{L_T^2} \ln\left(\frac{L}{L_T}\right), \quad L_T = \frac{2\pi}{\sqrt{k_B T}} \left(\frac{\kappa}{\sigma}\right)^{1/4}$$

$L_T$  is the thermal wavelength of flexural modes

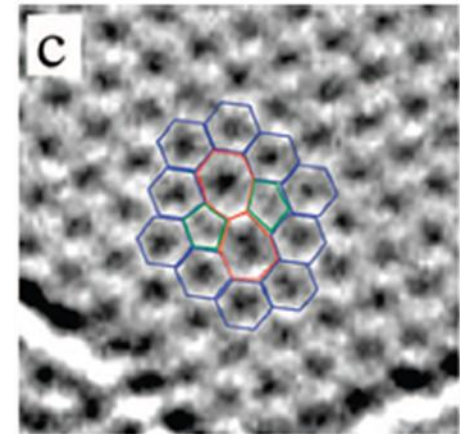
- **At T=300K,  $L \sim 1 \text{ \AA}$**
- **Indicating that free-floating graphene should always crumple at room temperature due to thermal fluctuations associated with flexural phonons**



# Topological Lattice Defects

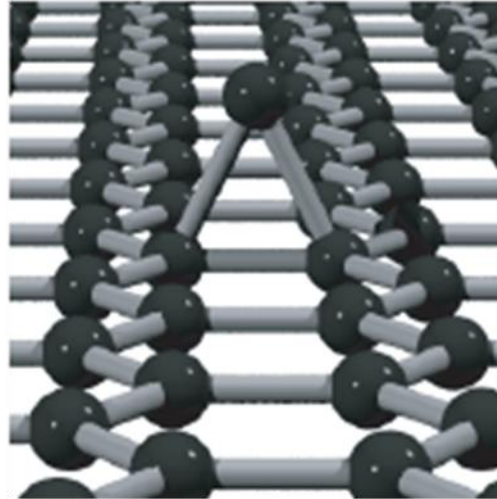
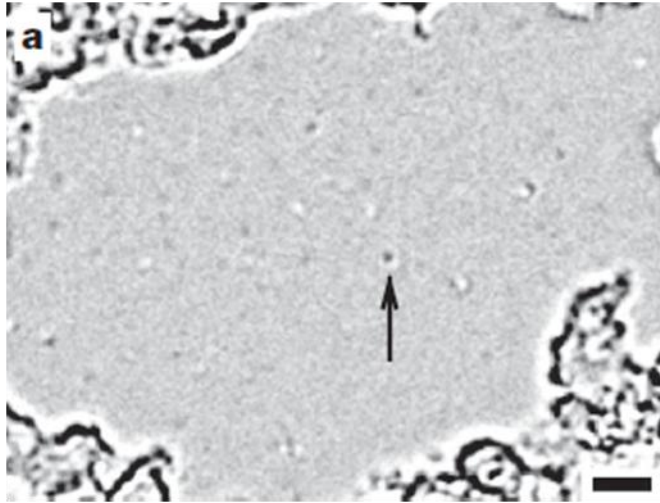


- Structural defects of the honeycomb lattice like **pentagons**, **heptagons**, and their combination such as **Stone-Wales defect** are named **topological lattice defect**.

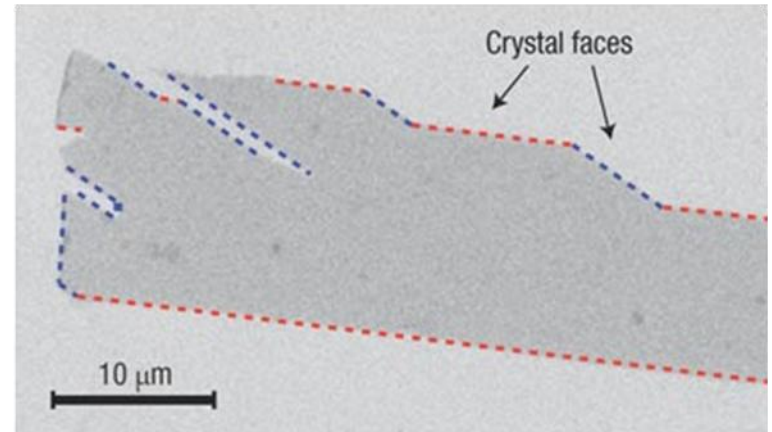
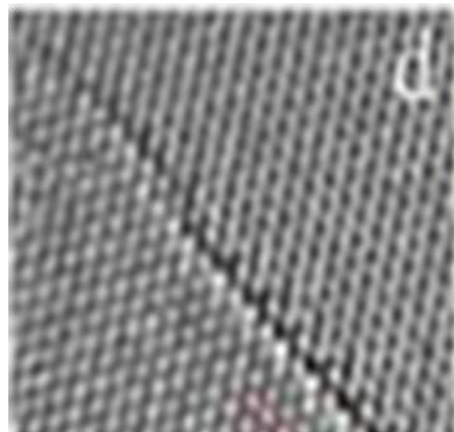


*Oleg V. Yazyev et al. PHYSICAL REVIEW B 81, 195420 2010;*  
*zhiyong wang et al. Acta Phys. Sin. Vol. 60, No. 1 (2011) 017102;*  
*Jannik C. Meyer et al. 2008 Vol. 8, No. 11 3582-3586*

# Ad-atoms & Cracks & Edges

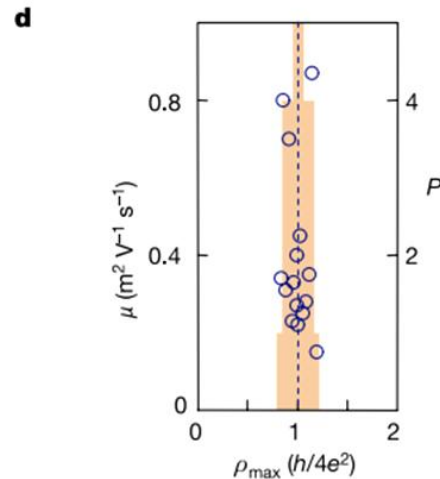
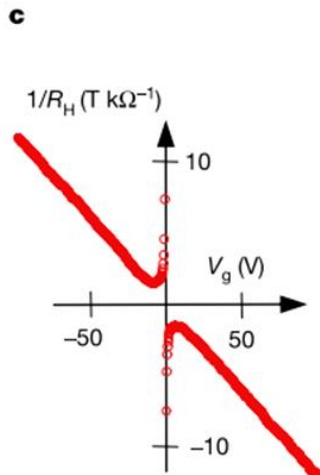
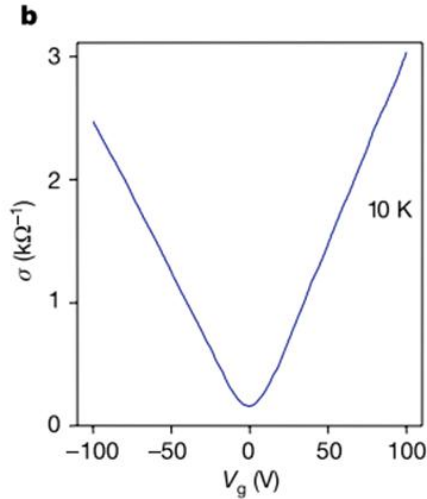
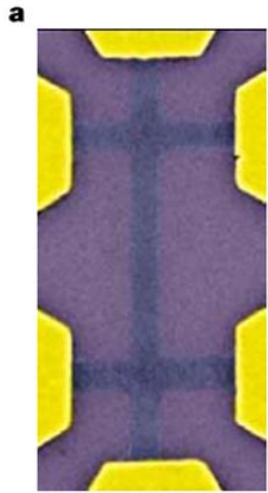


Above: ad-atoms.



Below: left(cracks)  
right(edges)

# Transport of Graphene



- **Conductance increases linearly with increasing carrier concentration.**  
(  $n = V_g \epsilon_0 \epsilon / t e$  )
- **Asymmetry of electron and hole**
- **Minimum conductivity near Dirac cone  $\sim 4e^2 / h$**

Novoselov, K. S. et al. Two-dimensional gas of massless Dirac fermions in graphene. Nature 438, 197–200 (2005).

# Far From the Dirac Cone

- **Scattering mechanics**

- random charged impurity centers (long range scattering)
- short-range scattering (e.g. defects).
- Phonon (no need considering)

- **Boltzmann transport theory**

- For  $n \gg n_i$ ,

$$-\mathbf{v}_k \cdot \nabla_{\mathbf{r}} f(\epsilon_k) - e(\mathbf{E} + \mathbf{v}_k \times \mathbf{H}) \cdot \nabla_{\mathbf{k}} f(\epsilon_k) = - \left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{\text{scatt}}$$

- **Coulomb potential scattering**

$$\sigma_{xx} = 2 \frac{e^2}{h} \frac{\mu^2}{u_0^2} = 2 \frac{e^2}{h} \frac{\pi v_F^2}{u_0^2} n,$$

- **Short-range scattering**

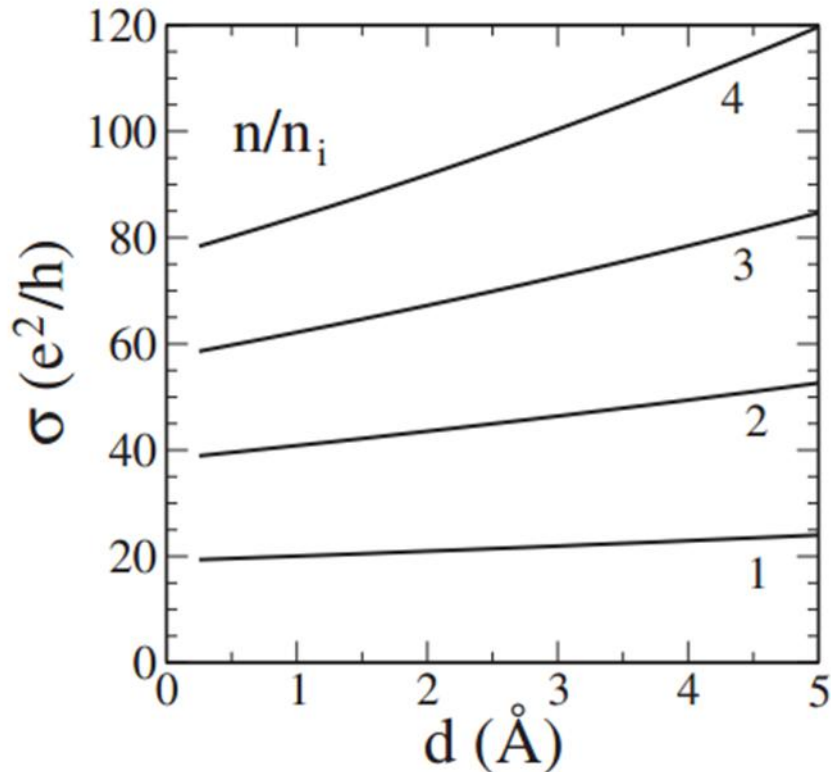
$$\sigma = C$$

**So, impurity scattering!**

Hwang, E. H., S. Adam, & S. Das Phys. Rev. Lett. 98, 186806 (2007).

Nomura K et.al. Phys. Rev. Lett, 2007, 98(7): 076602.

# Asymmetry of Electron and Hole



- **Approximate:**

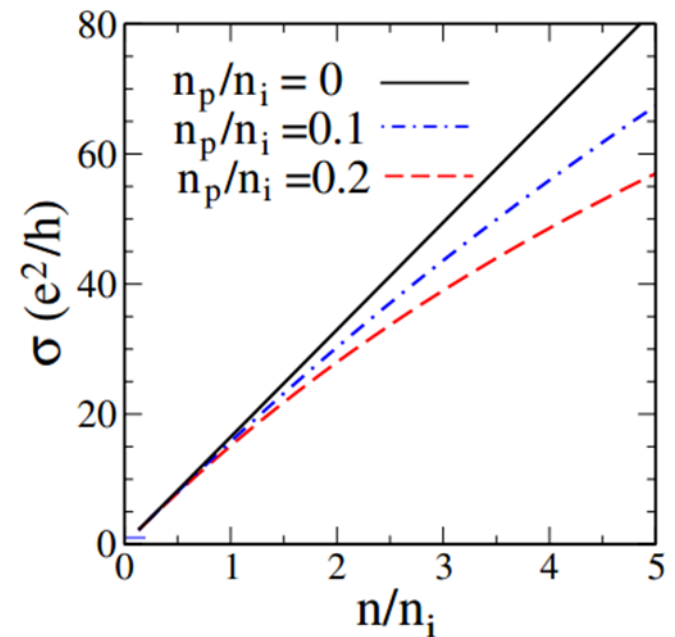
- **The impurities are confined in a 2D plane located at distance  $d$  from the interface.**

- **A small shift of  $d \sim 1-2 \text{ \AA}$**

- **Approximate that the impurities are confined in a 2D plane located at distance  $d$  from the interface**

# Sublinear for High Mobility Sample

- For high mobility sample, charged Impurity concentration  $n_i$  is small, the point defect play a more dominant role.
- Conductance corresponding to defect constant.
- For most sample  $n_p/n_i \ll 1$ , for high mobility samples,  $n_p/n_i \sim 0.2$





# Near Dirac Cone: Minimum Conductivity

- **Theoretically Prediction:**  $\sigma = \frac{4e^2}{\pi h}$
- **Experiment :**  $\sigma = \frac{4e^2}{h}$
- Intrinsic or extrinsic property of graphene?
- **Extrinsic :**
- **2D electrons and hole conducting puddles caused by impurities**
- **Considering screened impurity scattering,**

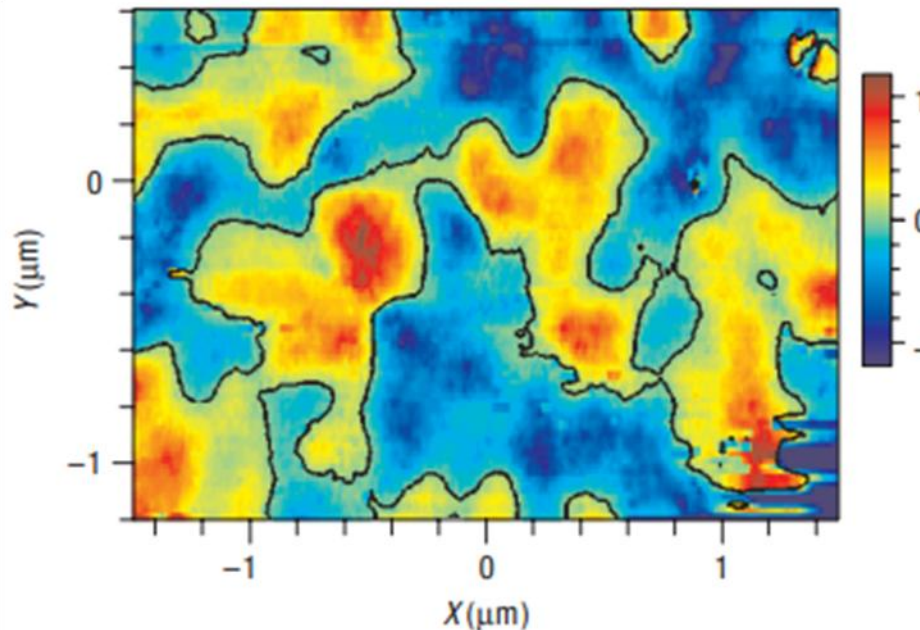
$$\sigma = \frac{e^2 n}{h n_{imp} G[2r_s]}$$

$$\frac{G[x]}{x^2} = \frac{\pi}{4} + 3x - \frac{3\pi x^2}{2} + \frac{x(3x^2 - 2)\arccos[1/x]}{\sqrt{x^2 - 1}},$$

- **For Graphene on SiO<sub>2</sub>:**  $\sigma = 20(e^2 / h)(n / n_{imp})$

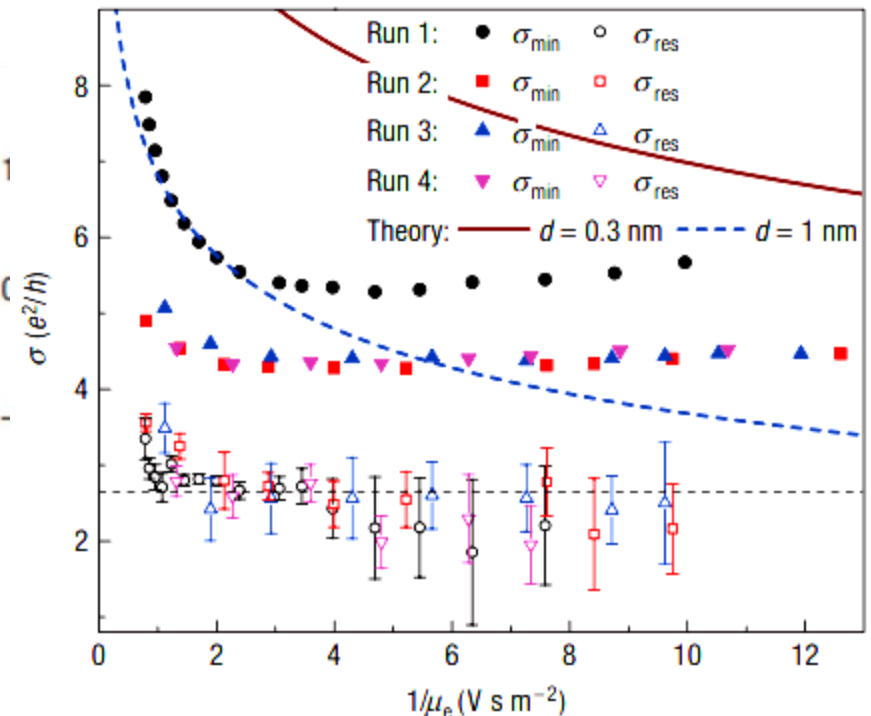
# Near Dirac Cone: Minimum Conductivity

- Observation of electron–hole puddles in graphene using a scanning single-electron transistor



J. Martin et al. Nature Physics, 4, 144 (2007).

- Experimental Verification



Chen J H, et al. Nature Physics, 2008, 4(5): 377-381.

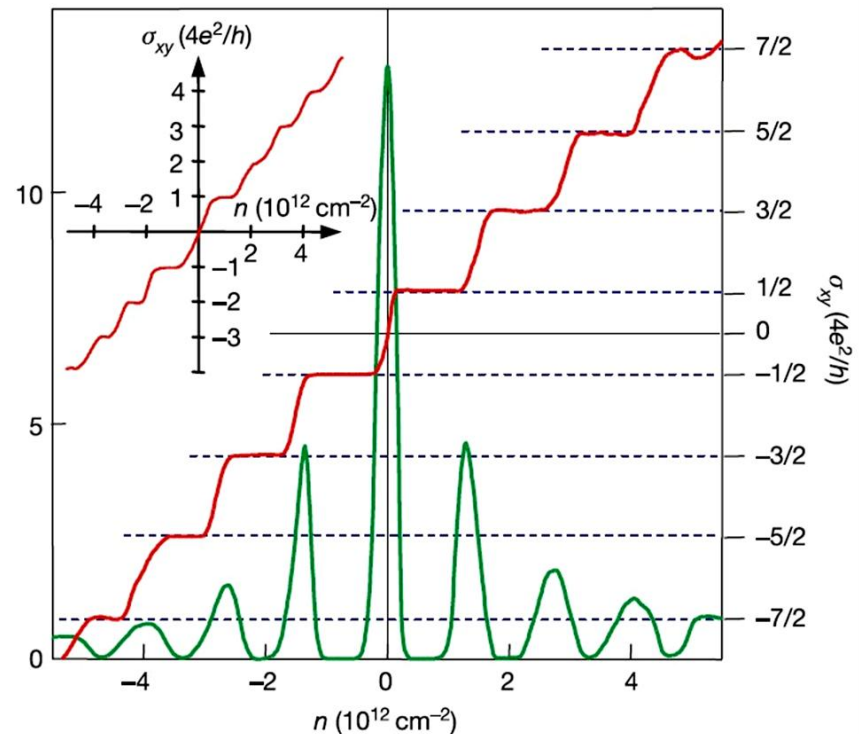
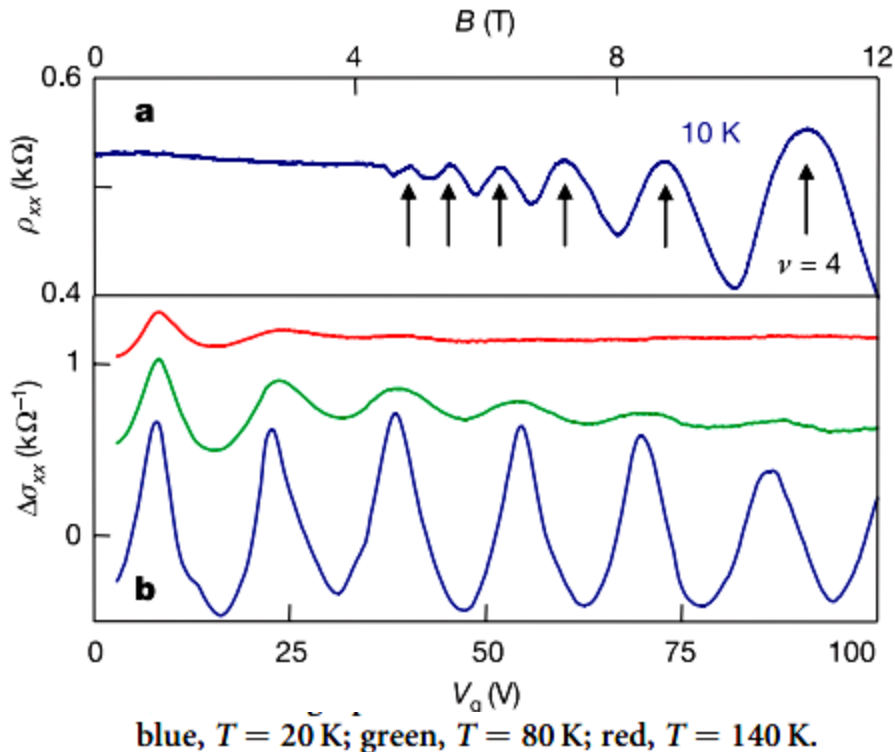
# Quantum Hall Effect

- **Half-integer Quantum Hall Effect (SDH Oscillation)**
- **Landau-Level Splitting in High Magnetism**
- **Fractional Quantum Hall Effect**

# Half-integer Quantum Hall Effect (SdHO)

## Two-dimensional gas of massless Dirac fermions in graphene

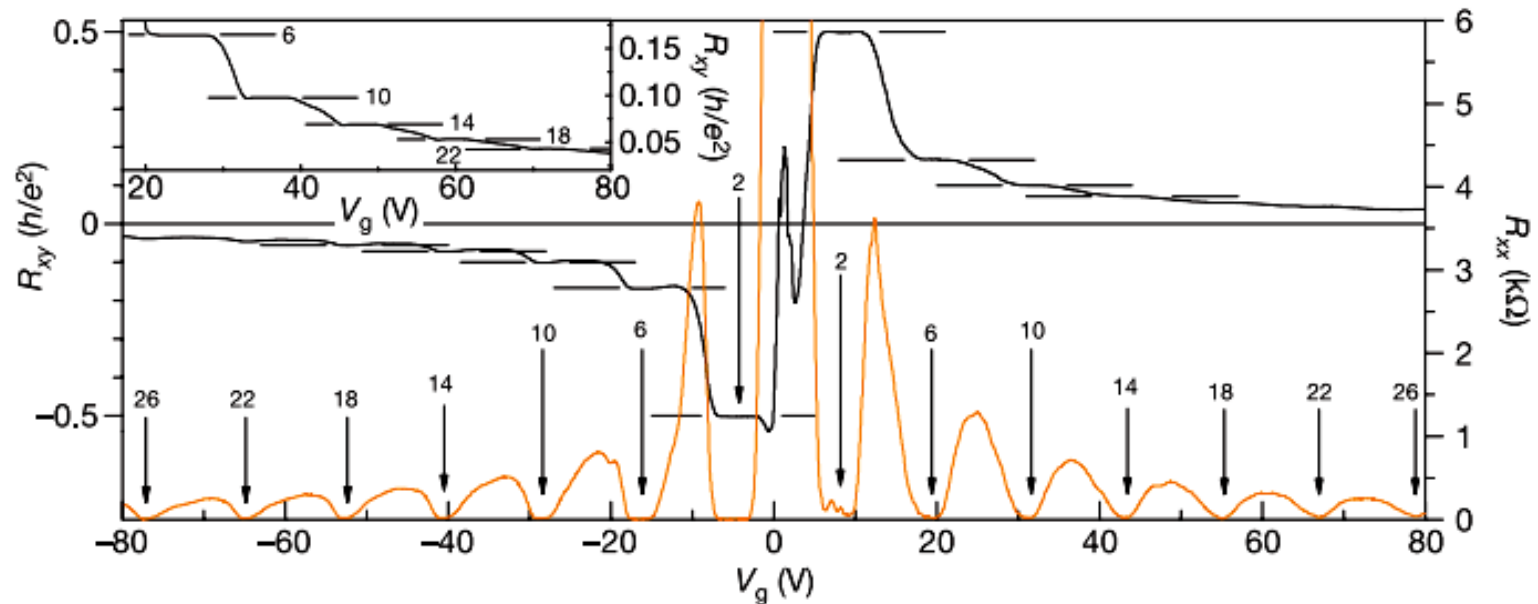
K. S. Novoselov<sup>1</sup>, A. K. Geim<sup>1</sup>, S. V. Morozov<sup>2</sup>, D. Jiang<sup>1</sup>, M. I. Katsnelson<sup>3</sup>, I. V. Grigorieva<sup>1</sup>, S. V. Dubonos<sup>2</sup> & A. A. Firsov<sup>2</sup>



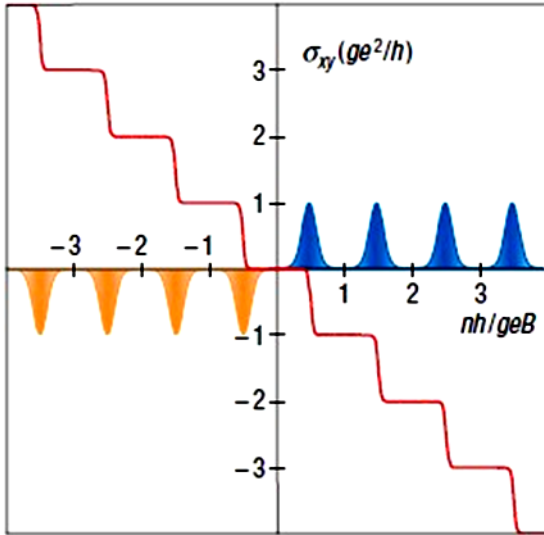
# Half-integer Quantum Hall Effect (SdHO)

## Experimental observation of the quantum Hall effect and Berry's phase in graphene

Yuanbo Zhang<sup>1</sup>, Yan-Wen Tan<sup>1</sup>, Horst L. Stormer<sup>1,2</sup> & Philip Kim<sup>1</sup>

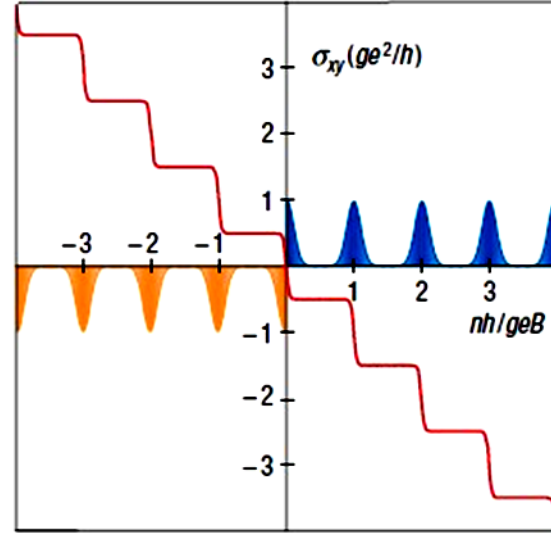


# Different Types of Landau quantization



2DEG

$$\sigma_{xy} = gn \frac{e^2}{h}$$



MLG

$\nu = \pm 2, \pm 6, \dots$

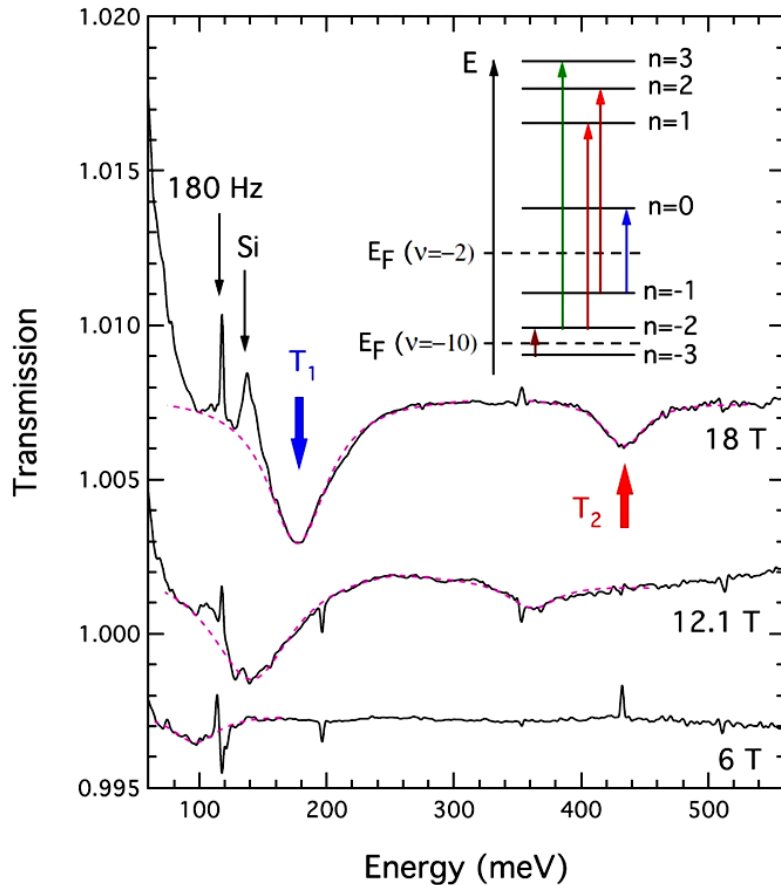
$$\sigma_{xy} = g \left( n + \frac{1}{2} \right) \frac{e^2}{h}$$

Landau Level:

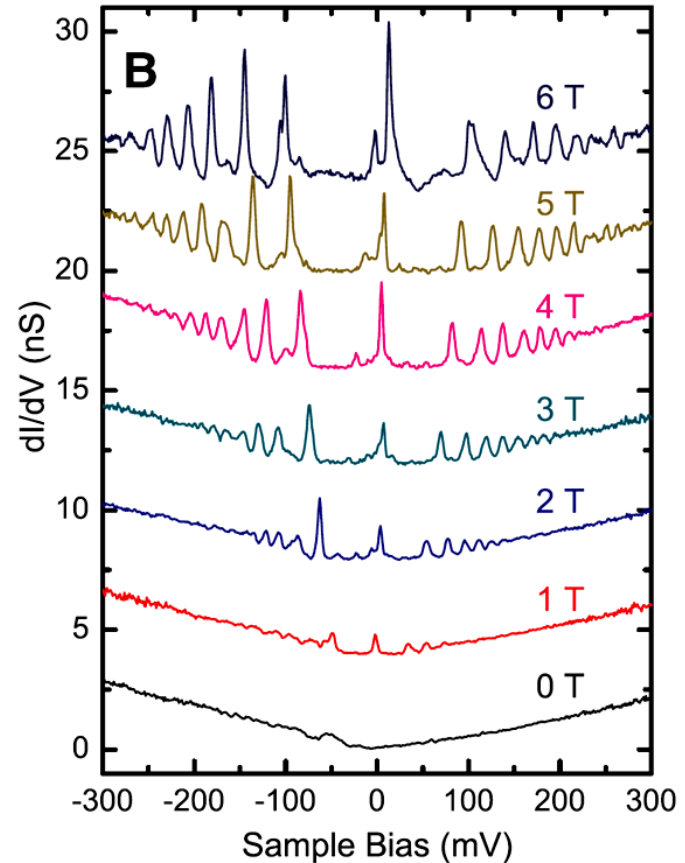
$$E_n = \text{sgn}(n) v_F \sqrt{2e\hbar B |n|}$$



# Observation of Landau Level in Graphene

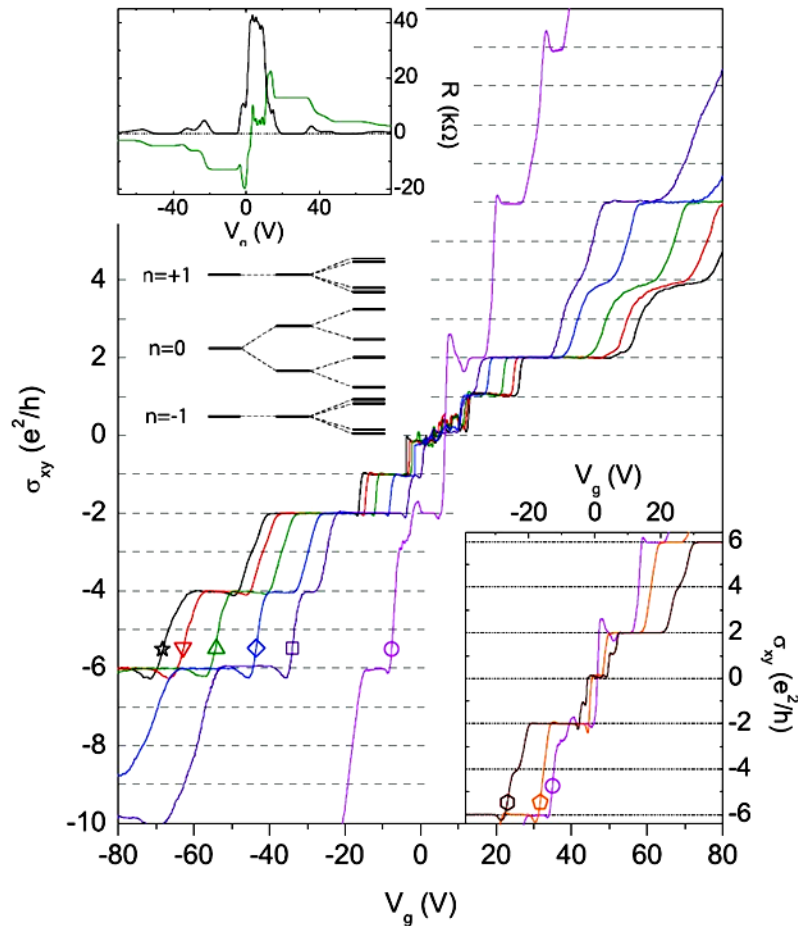


Infrared Spectroscopy of Landau Levels  
PRL 98, 197403 (2007)



STM Observation of LLs  
Miller et al., Science 324, 924 (2009)

# Symmetry Breaking: Landau-Level Splitting



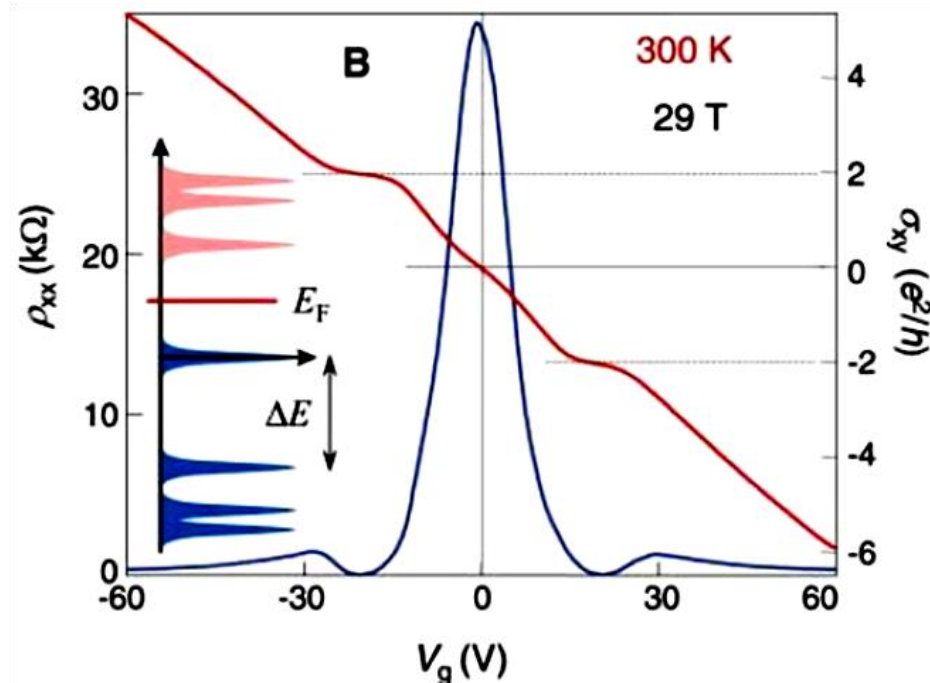
- **New sets:  $\nu = 0, \pm 1, \pm 4$**
- **$n=0$  LL splits into four sublevels, lifting spin and sub-lattice degeneracy**
- **$n=1$  LL, only the spin degeneracy is lifted**

# Room-Temperature QHE

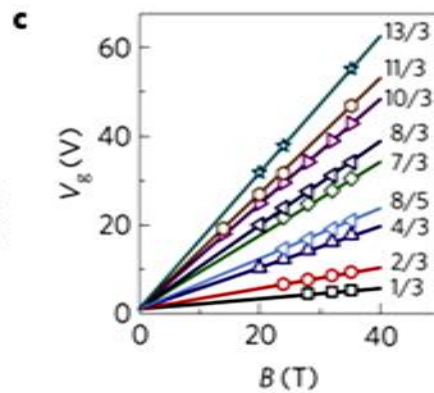
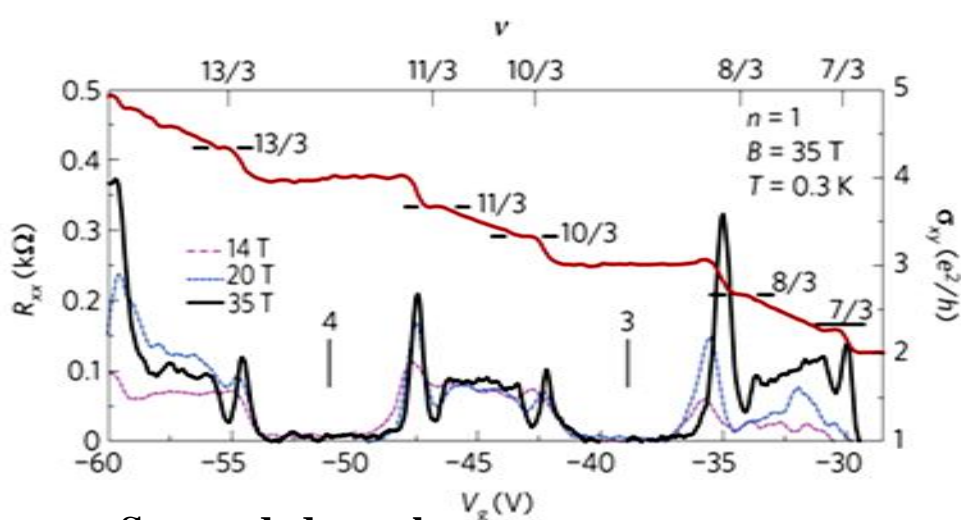
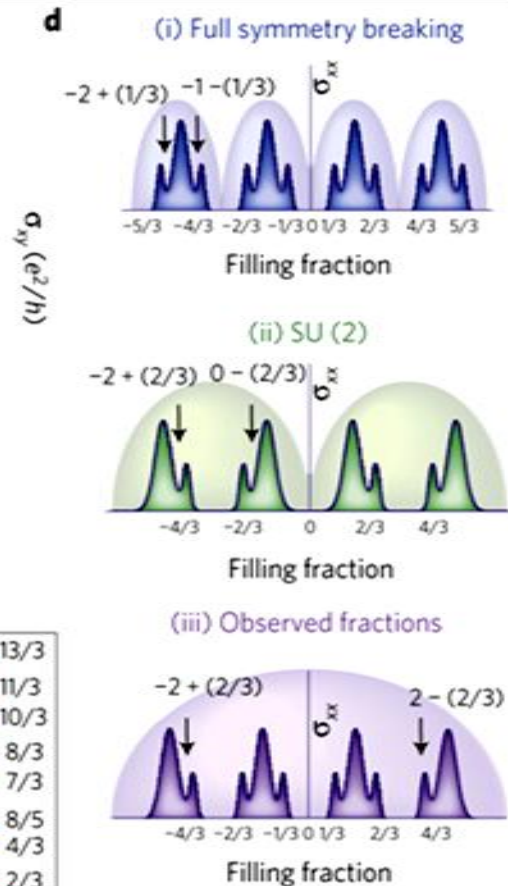
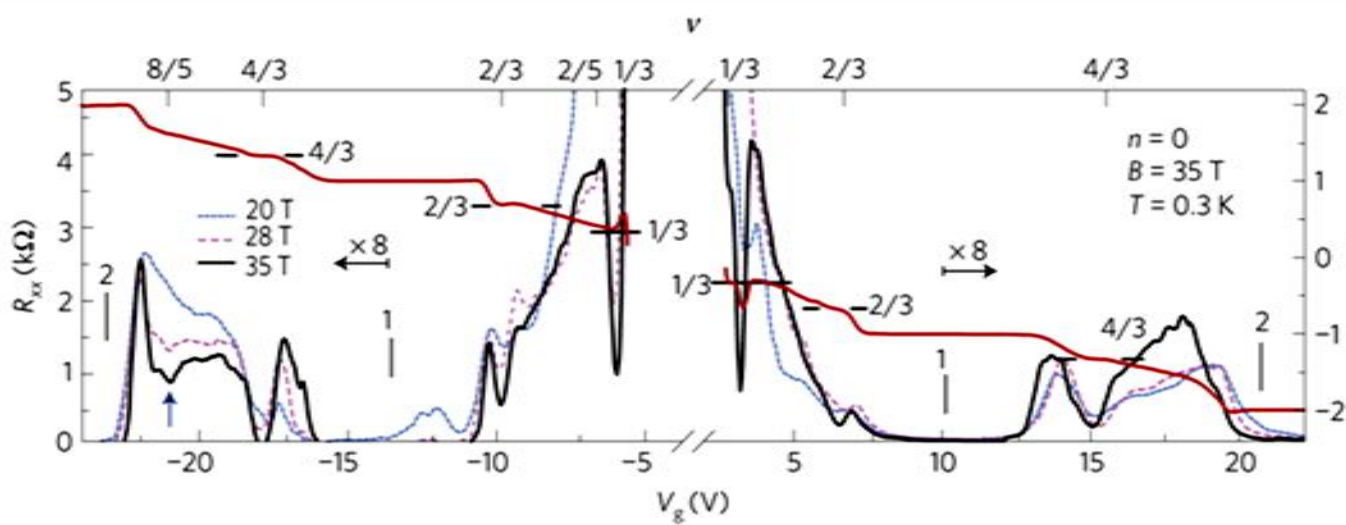
At 10T, cyclotron gaps  $\sim 1000\text{K}$  for graphene,  $\sim 10\text{K}$  (2DEG)

## Room-Temperature Quantum Hall Effect in Graphene

K. S. Novoselov,<sup>1</sup> Z. Jiang,<sup>2,3</sup> Y. Zhang,<sup>2</sup> S. V. Morozov,<sup>1</sup> H. L. Stormer,<sup>2</sup> U. Zeitler,<sup>4</sup> J. C. Maan,<sup>4</sup>  
G. S. Boebinger,<sup>3</sup> P. Kim,<sup>2\*</sup> A. K. Geim<sup>1\*</sup>



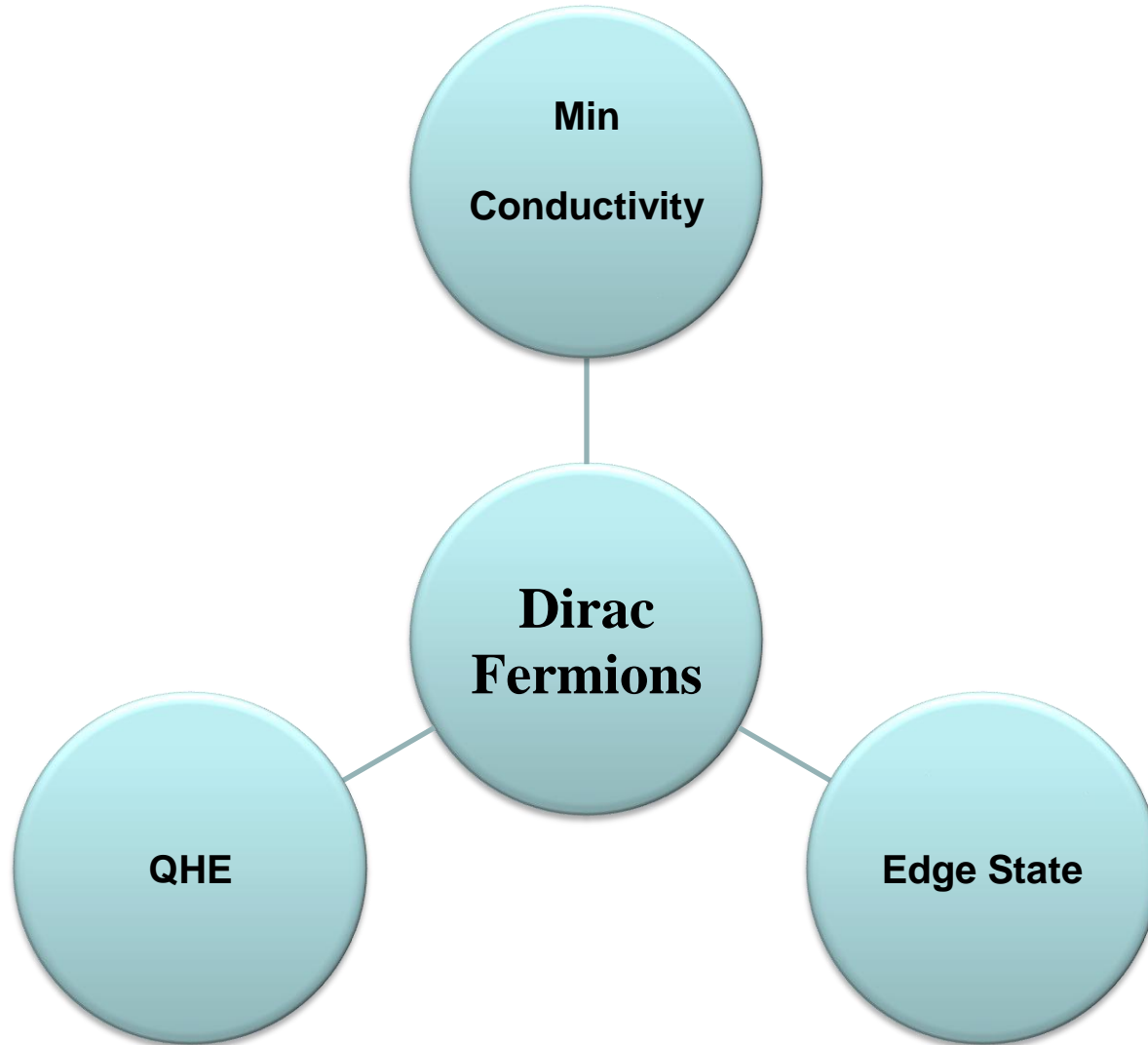
# Fractional Quantum Hall Effect



**Suspended graphene:**  
Nature 462, 196(2009) ; Nature 462, 192(2009)

**BN substrate:**  
Nature Physics ,7,693(2011)

# Summary



**Thank you!**